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Model For the Effect of Environmental Pollution on the Survival of Resource Dependent Population with Time Delay

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Introduction-

Large quantities of toxicants (pollutants) penetrate both the aquatic and terrestrial environments as a result of fast industrialization and technological advancement, severely impacting both the biological population and their resources. Toxicants disrupt resource metabolism when they come into contact with resource biomass. Because of this, the biomass's intrinsic growth could be negatively impacted. The resource-biomass can be more severely affected by toxicants than the consuming species. Forests, fisheries, fertile topsoil, crude oil, minerals, and other resources could be depleted as a result. According to Freedman and Shukla (1991), Shukla and Dubey (1997, 2009), and Shukla et al. (2009), species extinction can also happen in extreme instances, impacting the bio-diversity of the ecosystem. Consequently, research into the ways in which pollutants affect biological populations and the biomass resources upon which they rely is crucial. The resource-biomass may take some time to absorb the environmental toxin. Taking the delay impact into account in this case is, thus, quite reasonable and reflective of the actual system. The introduction of delay causes the system to behave unstablely, as is commonly observed.

With these considerations in mind, a delay differential model is suggested for investigating the impact of a toxicant on a biological population and resource biomass. The model presupposes that toxicants have an effect on the biomass of a biological population and its carrying capacity. Delays in toxicant absorption slow down resource biomass, which in turn impacts population dynamics in an indirect way. Both the local and global stability of the model have been investigated using numerical simulations and mathematical analysis.

Mathematical Model-

Here we take an ecosystem into consideration, where a pollutant threatens the very existence of a resource-dependent biological community. The logistic equation is believed to control the dynamics of population and resource-biomass. One possible representation of the above system of differential equations is:

$$\frac{dB}{dt} = r(N)B - \frac{r_0 B^2}{K(T)},$$
$$\frac{dN}{dt} = r_N(B)N - \frac{r_{N0} N^2}{M(T)}, \qquad (3.1.1)$$
$$\frac{dT}{dt} = Q(t) - \delta_0 T - \alpha_1 BT + \pi \upsilon BU + \theta_1 \delta_1 U,$$
$$\frac{dU}{dt} = -\delta_1 U + \alpha_1 BT - \upsilon BU + \theta_0 \delta_0 T.$$
$$B(0) \geq 0, \quad N(0) \geq 0, \quad T(0) \geq 0, \quad U(0) \geq 0.$$

In model (3.1.1), the variables denoted as, which represent the resource-biomass density, the biological population density, the concentration of the pollutant (toxicant) in the environment, and the concentration of the pollutant that the resource-biomass takes up from the environment, are utilized. Can be either constant, zero, or periodic, depending on how quickly the pollutant is introduced into the environment. is the rate at which a pollutant is removed from the environment, possibly through biological transformation, chemical hydrolysis, volatization, microbial degradation, or photosynthetic degradation; when a portion of the pollutant is absorbed, it may re-enter the resource biomass. is the rate of natural depletion that occurs as a result of the pollutant being consumed and then removed from the body; some of this pollutant may also be recycled and returned to the environment. In addition, resource biomass may cause a drop in the absorption concentration of the pollutant as the rate coefficient increases, and some of the dropout may re-enter the environment. is the pace at which a resource (biomass) depletes a pollutant (environmental pollution).

Here, we'll pretend that environmental pollutants don't impact resource-biomass growth per se, but that once they reach biomass, they transform into a toxin that slows down the growth rate of the resource-biomass. Depletion of the resource-biomass is postponed due to this conversion. So, here we are

$$W(t) = \int_{-\infty} \alpha f(t-p) U(s) ds \qquad (3.1.2)$$

Where, $\alpha > 0$ is a constant. $f(t)$ is a non-negative continuous function called a weight

Function or Kernel (Rao, 1992) and it satisfies,

$$\int_{-\infty}^{t} f(t-p)dp = g_0.$$

(3.1.3)

Where g_0 is a positive constant?

In equation (3.1.2), $W(t)$ may be thought as the concentration of toxic substance which has been formed by the conversion of $U(t)$ due to some metabolic changes. If we consider the weight function $f(t)$ in the form (Dubey and Hussain, [2004]):

$$f(t-p) = e^{-\alpha_0(t-p)}, \qquad \alpha_0 > 0.$$

Then equation (3.1.2) reduces to

$$W(t) = \int_{-\infty}^{t} \alpha \, e^{-\alpha_0(t-p)} U(s) ds.$$

Differentiate the above equation w.r.t't', we get

$$\frac{dW}{dt} = \alpha U - \alpha_0 W.$$

Thus, the inclusion of delay in the uptake of environment pollution by the resource leads to the system (3.1.1) as:

$$\frac{dB}{dt} = r(N,W)B - \frac{r_0 B^2}{K(T)},$$

$$\frac{dN}{dt} = r_N(B)N - \frac{r_{N0} N^2}{M(T)},$$

$$\frac{dT}{dt} = Q(t) - \delta_0 T - \alpha_1 BT + \pi \upsilon BU + \theta_1 \delta_1 U,$$

(3.1.4)

$$\frac{dU}{dt} = -\delta_1 U + \alpha_1 BT - \upsilon BU + \theta_0 \delta_0 T,$$

$$\frac{dW}{dt} = \alpha U - \alpha_0 W.$$

$B(0) \geq 0, N(0) \geq 0, T(0) \geq 0, U(0) \geq 0,$ and $W(0) \geq 0.$

One possible interpretation of in system (3.1.4) is the concentration of the poisonous substance that has been produced by a metabolic shift in the form of. The growth rate coefficient of the resource-biomass is assumed to be proportional to the concentration of the pollutant, while the carrying capacity is affected by the population. The natural depletion rate coefficient is denoted by and.

Model (3.1.4), is derived under following assumptions.

(H1): The function $r(N,W) > 0$, is the specific growth rate of the resource-biomass which decreases as the density of population and delay in the uptake of pollutant increases, i.e.,

$$r(0,0) = r_0 > 0,\ \frac{\partial r(N,W)}{\partial N} < 0,\ \frac{\partial r(N,W)}{\partial W} < 0 \quad \text{for } N \geq 0,\ W \geq 0.$$

(H2): The function $K(T)$ is the carrying capacity of the resource-biomass which the environment can support in the presence of environmental pollutant, and it decreases as the environmental concentration of the pollutant increases, i.e.,

$$K(0) = K_0 > 0,\ \ K'(T) > 0 \ \ \text{for } T > 0.$$

(H3): The function $r_N(B) > 0$ is the specific growth rate of biological population which increases as the density of resource-biomass increases, i.e.,

$$r_N(0) = r_{N0} > 0,\ \ r'_N(B) > 0 \ \ \text{for } B \geq 0.$$

(H4): The function $M(T)$ is the carrying capacity of the population which the environment can support in the presence of environmental pollutant, and it decreases as the environmental concentration of the pollutant increases, i.e.,

$$M(0) = M_0 > 0,\ \frac{dM}{dT} < 0,\ \text{for } T \geq 0.$$

We analyze model (3.1.4) for three values of $Q(t)$, namely,

To analyze the model (3.1.4), we need the bounds of dependent variable involved. For this we find the region of attraction in the following lemma.

Boundedness of Solution

Lemma (3.1.1): Suppose that assumptions (H1) - (H4) hold. Then all solutions of system (3.1.4) are bounded within the region Ω

Where $\theta_0 < 1$,

$\theta_1 < 1, \delta_1(1-\theta_1) > \alpha$ And $\delta = \min(\delta_0(1-\theta_0), \delta_1(1-\theta_1) - \alpha, \alpha_0)$.

$$\delta = \min(\delta_0(1-\theta_0), \delta_1(1-\theta_1) - \alpha, \alpha_0).$$

Proof: It is assumed here that all the initial values of variables considered in model (3.1.4) belongs to the region Ω and are positive.

From the first equation of system (3.1.4), we get

$$\frac{dB}{dt} \le r_0 B\left(1-\frac{B}{K_0}\right).$$

This implies that, $0 \le B \le K_0$.

From the second equation of model (3.1.4), we get

$$\frac{dN}{dt} \le r_{N0} N\left(1-\frac{N}{M_0}\right).$$

This implies that, $0 \le N \le M_0$.

From the third, fourth and fifth equation of model (3.1.4), we get

$$\frac{dT}{dt}+\frac{dU}{dt}+\frac{dW}{dt} \le Q_0 - \delta_0 T(1-\theta_0) - U(\delta_1(1-\theta_1)-\alpha) - \alpha_0 W$$

$$\le Q_0 - \delta(T+U+W),$$

Where $\delta = \min\left(\delta_0(1-\theta_0), \delta_1(1-\theta_1)-\alpha, \alpha_0\right)$.

This implies that, $0 \le T+U+W \le \frac{Q_0}{\delta}$.

Hence, the lemma follows.

The biological balance of the ecosphere is greatly influenced by forests. A forest provides many essential goods and services, including but not limited to: wood, energy, medicine, feed, and soil erosion prevention , microclimate and water cycle maintenance , and carbon dioxide sequestration . According to the World Bank, almost one billion people across the globe rely on forests for their primary medicinal needs . On top of that, 80–90% of the terrestrial biodiversity lives in forests. The eastern arc of the mountains between Tanzania and Kenya is home to 2000 km2 of woods that are home to around 121 endemic vertebrate species, making it the biodiversity hotspot with the highest ratio of endemics per area . Despite the forest's crucial functions, its resources are in danger from the relentless demands of human activity . Evidence suggests that human population expansion is substantial around the globe, which is putting a strain on forest resources. Most deforestation and habitat loss occurs because people require more land to build homes and cultivate food to support an ever-increasing human population . Poverty compounds the problem in emerging nations like Tanzania, where people rely heavily on land and forest resources for their livelihoods . Specifically, 90% of Tanzanian families use wood charcoal and firewood for energy, according to Shadrack et al.. It is safe to assume that Tanzania's forested areas are in danger of vanishing unless action is taken to lessen the reliance on forest resources. Similarly, according to The State of the World's Forests 2020 (FOSO), forests make up over 31% of the Earth's surface area . Nevertheless, 3.3 million hectares of forest lands were lost between 2010 to 2015]. The loss of 137 species every day is another consequence of deforestation, according to the estimate. This means that areas rich in biodiversity are likewise at risk when forests are threatened. Duncan argues that human lifestyles, industrial socioeconomic position all have a role in the depletion of forest resources. According to some estimates, the most important underlying factor in deforestation in Africa is people's socioeconomic condition. There is a lot of strain on forest lands because many African households rely on forest resources for survival and because farming techniques are inefficient owing to a lack of modern technology. According to Bhardwaj , there are two ways in which economic growth might impact deforestation: positively or negatively. The Immiserization Theory postulates this occurrence. Impoverished people, according to the argument, degrade forests because they take more natural resources to satisfy their demands. The results of Raphael et al.] corroborate the notion, showing that households with fewer assets rely more on income from forests than other households. Growth in the economy, on the other hand, halts deforestation because it generates non-farm jobs, gives people more agency in shaping forest management policies, and raises public awareness about the need of protecting forests. The number of individuals living on $1.90 USD or less per day has decreased from 1.9 billion in 1990 to 736 million in 2015, according to World Bank estimates, indicating a remarkable decline in extreme poverty globally . However, the rate of extreme poverty is rising sharply across Sub-Saharan Africa. An increasing number of households are living below the poverty line as a result of the failure of poverty reduction efforts in Africa, especially in Tanzania, to keep pace with population growth. The forecast indicates that by 2030, 90% of the world's poorest people will be residing in Sub-Saharan Africa, which poses a threat to the region's forests. In Tanzania, 84.4% of the poor reside in rural areas, while 15.7% call urban areas home, according to a breakdown of the poor by geographic domain. Bhardwaj argues that poverty and environmental degradation are both caused by the unsustainable exploitation of natural resources. Therefore, there have been many beneficial outcomes from researching the consequences of population growth on forest resources, including improved methods of poverty reduction and environmental management. People in underdeveloped nations rely heavily on trees for their livelihoods, despite the difficulties. Thus, it is impossible to try to restrict people's use of forest resources in any way. Nonetheless, it is possible to control harvesting and usage by managing forest resources. The theoretical and experimental effects of population increase and related pressures on forest biomass have been investigated by several researchers and references therein). Research like this aims to fill gaps in our knowledge about how forest biomass and human populations interact. The primary concerns were as follows: how does the mechanism that controls the interaction between different kinds of humans, forests, and wildlife change over the course of a lifetime? To what extent are the system's primary parameters defined? When trees are cut down, how quickly can their natural resources grow back? How can we best preserve these valuable assets? And Can we trust the model that was created? Understanding the variables and relationships that create complexity in human-forest interactions and making predictions about the behaviour of the system produced can be achieved through the application of theoretical methodologies under dynamic system models. This review increases our knowledge of these approaches. Since it would be impossible to incorporate every article in the review, we followed the steps outlined by Meline [33] to choose which studies to include. In particular, the items could not have been included unless they fulfilled the following criteria: The paper has been published in a journal that is listed in the Web of Science. It contains at least two keywords and discusses dynamical systems. The following is the outline for the remainder of the review: Section 2 details the methods utilised for the systematic review, and Section 3 offers and analyses a number of mathematical models that fulfilled the inclusion and exclusion criteria. Section 4 presents the debate and conclusion based on Section 3's findings.

Global Stability-

Theorem (3.4.2): In addition to the assumption (H1) – (H4), let $r(N,W)$, $r_N(B)$, $K(T)$ and $M(T)$ satisfy the conditions

$$0 \le -\frac{\partial r(N,W)}{\partial N} \le \rho_1, \quad 0 \le -\frac{\partial r(N,W)}{\partial W} \le \rho_2,$$

$$0 \le r_N'(B) \le \rho_3, \quad K_m \le K(T) \le K_0, \quad 0 \le -K'(T) \le k, \quad M_m \le M(T) \le M_0,$$

$$0 \le -M'(T) \le m.$$

(3.4.6)

In Ω for some positive constants $\rho_1, \rho_2, \rho_3, k, K_0, m, M_0$. Then if the following inequalities hold

$$(\rho_1 + \rho_3)^2 < \frac{1}{2}\frac{r_0}{K(T^*)}\frac{r_{N0}}{M(T^*)},$$

(3.4.7)

$$\left(r_{N0} M_0 \frac{m}{M_m^2}\right)^2 < \frac{2}{3}\frac{r_{N0}}{M(T^*)}(\delta_0 + \alpha_1 B^*),$$

(3.4.8)

$$\left(r_0 K_0 \frac{k}{K_m^2} + \alpha_1 \frac{Q_0}{\delta} - \pi\upsilon K_0\right)^2 < \frac{1}{3}\frac{r_0}{K(T^*)}(\delta_0 + \alpha_1 B^*), \tag{3.4.9}$$

$$\left((\alpha_1 - \upsilon)\frac{Q_0}{\delta}\right)^2 < \frac{1}{3}\frac{r_0}{K(T^*)}(\delta_1 + \upsilon B^*),$$

(3.4.10) $$((\pi\upsilon + \alpha_1)B^* + \theta_0\delta_0 + \theta_1\delta_1)^2 < \frac{4}{9}(\delta_0 + \alpha_1 B^*)(\delta_1 + \upsilon B^*),$$

(3.4.11) $$\alpha^2 < \frac{2}{3}\alpha_0(\delta_1 + \upsilon B^*),$$

(3.4.12) $$\rho_2^2 < \frac{1}{2}\alpha_0\frac{r_0}{K(T^*)},$$

(3.4.13)

E^* is globally asymptotically stable with respect to all solutions initiating in the positive orthant Ω.

Proof: Consider the following positive definite function about E^*

$$V(B,N,T,U,W)=\left(B-B^{*}-B^{*}\ln\frac{B}{B^{*}}\right)+\left(N-N^{*}-N^{*}\ln\frac{N}{N^{*}}\right)+\frac{1}{2}(T-T^{*})^{2}$$
$$+\frac{1}{2}(U-U^{*})^{2}+\frac{1}{2}(W-W^{*})^{2}.$$

Differentiating V with respect to time t, we get

$$\frac{dV}{dt}=\left(\frac{B-B^{*}}{B}\right)\frac{dB}{dt}+\left(\frac{N-N^{*}}{N}\right)\frac{dN}{dt}+(T-T^{*})\frac{dT}{dt}+(U-U^{*})\frac{dU}{dt}+(W-W^{*})\frac{dW}{dt}.$$

Substituting values of $\frac{dB}{dt},\frac{dN}{dt},\frac{dT}{dt},\frac{dU}{dt}$ and $\frac{dW}{dt}$ from the system of equation (3.1.4) in the above equation and after doing some algebraic manipulations and considering functions,

$$\eta_{1}(N,W)=\begin{cases}\dfrac{r(N,W)-r(N^{*},W)}{N-N^{*}}, & N\neq N^{*},\\ \dfrac{\partial r(N^{*},W)}{\partial N}, & N=N^{*},\end{cases} \tag{3.4.14}$$

$$\eta_{2}(N^{*},W)=\begin{cases}\dfrac{r(N^{*},W)-r(N^{*},W^{*})}{W-W^{*}}, & W\neq W^{*},\\ \dfrac{\partial r(N^{*},W^{*})}{\partial W}, & W=W^{*}\end{cases} \tag{3.4.15}$$

$$\eta_{3}(T)=\begin{cases}\dfrac{\dfrac{1}{K(T)}-\dfrac{1}{K(T^{*})}}{T-T^{*}}, & T\neq T^{*},\\ -\dfrac{K'(T^{*})}{K^{2}(T^{*})}, & T=T^{*},\end{cases} \tag{3.4.16}$$

$$\xi_{1}(B)=\begin{cases}\dfrac{r_{N}(B)-r_{N}(B^{*})}{B-B^{*}}, & B\neq B^{*},\\ r'(B^{*}), & B=B^{*},\end{cases}$$

(3.4.17)

$$\xi_{2}(T)=\begin{cases}\dfrac{\dfrac{1}{M(T)}-\dfrac{1}{M(T^{*})}}{T-T^{*}}, & T\neq T^{*},\\ -\dfrac{M'(T^{*})}{M^{2}(T^{*})}, & T=T^{*},\end{cases}$$

(3.4.18)

we get

$$\frac{dV}{dt} = -\frac{1}{4}a_{11}(B-B^*)^2 + a_{12}(B-B^*)(N-N^*) - \frac{1}{2}a_{22}(N-N^*)^2,$$

$$= -\frac{1}{2}a_{22}(N-N^*)^2 + a_{23}(N-N^*)(T-T^*) - \frac{1}{3}a_{33}(T-T^*)^2,$$

$$= -\frac{1}{3}a_{33}(T-T^*)^2 + a_{31}(T-T^*)(B-B^*) - \frac{1}{4}a_{11}(B-B^*)^2,$$

$$= -\frac{1}{4}a_{11}(T-T^*)^2 + a_{14}(B-B^*)(U-U^*) - \frac{1}{3}a_{44}(U-U^*)^2,$$

$$= -\frac{1}{3}a_{44}(U-U^*)^2 + a_{43}(U-U^*)(T-T^*) - \frac{1}{3}a_{33}(T-T^*)^2,$$

$$= -\frac{1}{3}a_{44}(U-U^*)^2 + a_{45}(U-U^*)(W-W^*) - \frac{1}{2}a_{55}(W-W^*)^2,$$

$$= -\frac{1}{2}a_{55}(W-W^*)^2 + a_{51}(W-W^*)(B-B^*) - \frac{1}{4}a_{11}(B-B^*)^2.$$

where

$$a_{11} = \frac{r_0}{K(T^*)}, \quad a_{12} = \eta_1(N,W) + \xi_1(B), \quad a_{22} = \frac{r_{N0}}{M(T^*)}, \quad a_{23} = -r_{N0}N\xi_2(T),$$

$$a_{13} = -(r_0 B\eta_3(T) + \alpha_1 T - \pi\upsilon\, B), \;\; a_{14} = \alpha_1 T - \upsilon U, \;\; a_{15} = \eta_2(N^*,W), \;\; a_{44} = \delta_1 + \upsilon B^*,$$

$$a_{34} = \pi\upsilon\, B^* + \theta_1\delta_1 + \alpha_1 B^* + \theta_0\delta_0, \quad a_{45} = \alpha, \quad a_{33} = \delta_0 + \alpha_1 B^*, \quad a_{55} = \alpha_0.$$

The sufficient condition for $\frac{dV}{dt}$ to be negative definite are that the following inequalities hold

$$a_{12}^2 < \frac{1}{2}a_{11}a_{22}, \quad a_{23}^2 < \frac{2}{3}a_{22}a_{33}, \quad a_{13}^2 < \frac{1}{3}a_{11}a_{33}, \quad a_{14}^2 < \frac{1}{3}a_{11}a_{44}, \quad a_{34}^2 < \frac{4}{9}a_{33}a_{44},$$

Now, from equation (3.4.6) and mean value theorem, we note that

$$|\eta_1(N,W)| \le \rho_1, \; |\eta_2(N^*,W)| \le \rho_2, \; |\eta_3(T)| \le \frac{k}{K_m^2}, \; |\xi_1(B)| \le \rho_3, \; |\xi_2(T)| \le \frac{m}{M_m^2}. \quad (3.4.20)$$

Further, we note that the stability conditions (3.4.7) - (3.4.13) as stated in theorem (3.4.2), can be obtained by maximizing the left-hand side of inequalities (3.4.19). This completes the proof of theorem (3.4.2).

Case II: When $Q(t)=0$, i.e., in the case of instantaneous emission of the pollutant into the environment, the corresponding results can be obtained from case I by substituting $Q_0 = 0$. In particular, it is noted that the resource-biomass and the population settle down to their respective equilibrium levels under certain conditions, and their magnitude are greater than their respective magnitude in case I.

Case III: Periodic introduction of the pollutant into the environment, i.e., $Q(t) = Q_0 + \varepsilon\phi(t),\ \phi(t+\omega) = \phi(t)$.

In this case, model (3.1.4) can be written in the vector matrix form as

$$\frac{dx}{dt} = A(x) + \varepsilon C(t), \quad \mathbf{x}(0) = \mathbf{x}_0,$$

(3.4.21)

Where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} B \\ N \\ T \\ U \\ W \end{bmatrix}, \mathbf{x}_0 = \begin{bmatrix} B(0) \\ N(0) \\ T(0) \\ U(0) \\ W(0) \end{bmatrix}, C(\mathrm{t}) = \begin{bmatrix} 0 \\ 0 \\ \phi(\mathrm{t}) \\ 0 \\ 0 \end{bmatrix},$$

$$A(x) = \begin{bmatrix} r(x_2, x_5)x_1 - \dfrac{r_0 x_1^2}{K(x_3)} \\ r_N(x_1)x_2 - \dfrac{r_{N0} x_2^2}{M(x_3)} \\ Q_0 - \delta_0 x_3 - \alpha_1 x_1 x_3 + \pi \upsilon x_1 x_4 + \theta_1 \delta_1 x_4 \\ -\delta_1 x_4 + \alpha_1 x_1 x_3 - \upsilon x_1 x_4 + \theta_0 \delta_0 x_3 \\ \alpha x_4 - \alpha_0 x_5 \end{bmatrix}.$$

Let M^* be the variational matrix corresponding to the positive equilibrium $E^*(B^*, N^*, T^*, U^*, W^*)$ Then under an analysis similar to Freedman and Shukla [1991], we can state the following two theorems.

Theorem (3.4.3): If M^* has no eigenvalues with zero real parts, then system (3.1.4) with $Q(t) = Q_0 + \varepsilon\phi(t),\ \phi(t+\omega) = \phi(t)$ has a periodic solution of period $\omega, (B(t,\varepsilon),$ $N(t,\varepsilon), T(t,\varepsilon), U(t,\varepsilon), W(t,\varepsilon))$ such that $(B(t,0),\ N(t,0),\ T(t,0),\ U(t,0),\ W(t,0)) =$

Theorem (3.4.4): If M^* has no eigenvalues with zero real parts, then for sufficiently

Small ε, the stability behavior of system (3.1.4) is same as that of

Moreover, a periodic solution up to order ε can be computed as

$$x(t,\varsigma,\varepsilon)=\bar{x}+e^{\overset{*}{M}t}\left[\int_0^t e^{\overset{*}{M}s}C(s)ds-\left(e^{\overset{*}{M}\omega}-I\right)^{-1}e^{\overset{*}{M}\omega}\int_0^{\omega}e^{\overset{*}{M}s}C(s)ds\right]\varepsilon+O(\varepsilon).$$

The above results imply that a small periodic influx of pollutant causes a periodic behavior in the system.

Conclusion-

In order to evaluate the effects of resource-biomass dependent environmental contamination with time delay, this chapter proposes and analyses a non-linear mathematical method. It is believed that biomass is a resource that partially determines population density. Stability theory of ordinary differential equations has been used to thoroughly analysis the model. Under some circumstances, the nontrivial equilibrium is shown to be analytically and visually asymptotically stable in an equilibrium analysis. The magnitude of the population's equilibrium level grows with increasing resource-biomass density and decreases with increasing toxicant concentration into the environment, as demonstrated in figure (1) of chapter 2, when pollutants are continuously introduced into the environment. Our model does not directly account for the effect of pollutant concentration on population or resource-biomass growth rates. The growth rate of the resource biomass is affected by metabolic changes that convert this pollutant into another chemical toxicant when it is absorbed by the biomass. As population density rises, resource biomass is likewise growing at a slower rate. Time lag caused by chemical toxicant synthesis reduces equilibrium level of resource-biomass, as shown in the analysis Perhaps the concentration of the pollutant was insufficient to deplete the resource-biomass and population in the case of the immediate introduction of the pollutant into the ecosystem. So, the pollutant will be fully washed out and the population and resource-biomass would recover to their original carrying capacities, with densities that are larger than what they would be in the case of continuous pollutant introduction. It has been also noted that a small periodic introduction of pollutant into the environment induces a periodic behavior in the system.

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